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EXPERIMENTAL DETERMINATION OF THE YAWING MOMENT DUE TO YAWING CONTRIBUTED BY THE WING, FUSELAGE, AND VERTICAL TAIL OF A MIDWING AIRPLANE MODEL

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EXPERIMENTAL DETERMINATION OF THE YAWING MOMENT DUE TO YAWING CONTRIBUTED BY THE WING, FUSELAGE, AND VERTICAL TAIL OF A MIDWING AIRPLANE MODEL

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SUMMARY

Values of the lateral-stability derivative C_{n_r}, the rate of change of yawing-moment coefficient with yawing angular velocity, contributed by the wing, the fuselage, and the vertical tail have been determined for a midwing airplane model by the free-oscillation method.

It was found that the values of C_{n_r} contributed by the vertical tail and by the profile drag of the wing were in good agreement with theory. The damping contributed by the wing varied as the square of the lift coefficient, but the actual values were somewhat lower than those predicted by existing theory. The value of C_{n_r} contributed by the fuselage appeared to be negligible.

An empirical formula is presented for obtaining an approximate value of C_{n_r} for a conventional midwing airplane.

INTRODUCTION

In calculating the lateral stability of an airplane, difficulty is often experienced in estimating values of the stability derivative C_{n_r}, the rate of change of yawing-moment coefficient with yawing angular velocity. Although theoretical methods for obtaining the value of C_{n_r} contributed by the vertical tail and the wing are given in references 1, 2, and 3, little recent experimental work has been done to determine values of this derivative. In order to provide experimental data on the contributions of the wing, the fuselage, and the vertical tail to C_{n_r}, some measurements for a midwing airplane model have been made in the NACA free-flight tunnel. Additional measurements were made for a rectangular wing of high-lift section in order to extend the lift coefficients to the high values encountered by full-scale airplanes. The results are presented in the present report.

A free-oscillation method similar to that described in reference 4 was used. The values of C_{n_r} were directly determined from the damping of free-yawing oscillations, which were obtained with the models mounted on a strut that permitted freedom only in yaw.

SYMBOLS

C_{n_r} rate of change of yawing-moment coefficient with yawing angular velocity per unit of $rb/2V$ $\left[\partial C_n/\partial\left(\frac{rb}{2V}\right)\right]$

C_{n_β} rate of change of yawing-moment coefficient with angle of sideslip $(\partial C_n/\partial\beta)$

C_L lift coefficient (L/qS)

C_{L_w} lift coefficient of wing alone

ΔC_{L_f} increment of lift coefficient due to flap

C_{D_o} profile-drag coefficient (D_o/qS)

$C_{D_{o_w}}$ profile-drag coefficient of wing alone

$\Delta C_{D_{o_f}}$ increment of profile-drag coefficient due to flap

C_n yawing-moment coefficient (N/qbS)

N yawing moment, foot-pounds

N_r rate of change of aerodynamic yawing moment with yawing angular velocity $(\partial N/\partial r)$

N_{r_f} rate of change of frictional yawing moment with yawing angular velocity $[(\partial N/\partial r)_f]$

N_ψ rate of change of aerodynamic yawing moment with angle of yaw $(\partial N/\partial\psi)$

k	rate of change of restoring moment of torsion spring with angle of yaw
L	lift, pounds
D_o	profile drag, pounds
q	dynamic pressure, pounds per square foot $\left(\frac{1}{2}\rho V^2\right)$
S	wing area, square feet
r	yawing angular velocity, radians por second
b	wing span, feet
b_f	flap span, feet
V	airspeed, foot per second
ρ	air density, slugs per cubic foot
ψ	angle of yaw, radians
ψ_{max_o}	maximum amplitude of yawing oscillation at zero time, radians
ψ_{max_t}	maximum amplitude of yawing oscillation at time t, radians
β	angle of sideslip, radians $(-\psi)$
a	total logarithmic decrement or damping factor
a_f	logarithmic decrement due to friction
t	time, seconds
T	period of yawing oscillation, seconds
A	aspect ratio
λ	taper ratio (ratio of tip chord to root chord)
l	distance from center of gravity to rudder hinge line, feet
I_Z	yawing moment of inertia, slug-feet square
k_o, k_1, k_2, k_3, k_f	constants

METHOD

The equation of motion of a system having freedom only in yaw can be expressed, to a close first approximation, as

$$I_Z \frac{d^2\psi}{dt^2} - \left(\overset{-N_r}{N_{r_\Lambda}} + N_{r_f}\right)\frac{d\psi}{dt} - \left(\overset{-N_\psi}{N_{\psi_\Lambda}} + k\right)\psi = 0 \qquad (1)$$

The yawing motion of the system represented by this equation can be expressed by an equation of the form

$$\psi = e^{-at}\ (A \sin bt + B \cos bt)$$

which represents a damped harmonic oscillation where the ratio of the maximum amplitudes of successive oscillations is

$$\frac{\psi_{max_t}}{\psi_{max_o}} = e^{-at}$$

The value of a, the logarithmic decrement or the damping factor, can be determined from the experimentally recorded angles of yaw against time by means of this relationship which, when transposed, gives

$$a = \frac{\log \psi_{max_o} - \log \psi_{max_t}}{t} \qquad (2)$$

The damping derivative expressed in terms of the damping factor is

$$N_r + N_{r_f} = -2I_Z a \qquad (3)$$

and the damping derivative due to friction is

$$N_{r_f} = -2I_Z a_f \qquad (4)$$

Combining equations (3) and (4) gives

$$N_r = -2I_Z\ (a - a_f)$$

or, in nondimensional form,

$$C_{n_r} = -\frac{4I_Z V}{qSb^2}\ (a - a_f) \qquad (5)$$

The period of the yawing oscillation expressed in terms of the coefficients of equation (1) is

$$T = \frac{2\pi}{\sqrt{\left(\frac{N_r + N_{r_f}}{2I_Z}\right)^2 - \left(\frac{N_\psi + k}{I_Z}\right)}} \qquad (6)$$

The effect of friction on the period is negligible. At zero airspeed, when N_r and N_ψ become zero, equation (6) reduces to

$$T = \frac{2\pi}{\sqrt{\frac{-k}{I_Z}}}$$

or

$$I_Z = -\frac{kT^2}{4\pi^2} \qquad (7)$$

By substituting in equation (7) the value of T at zero airspeed, the yawing moment of inertia I_Z can be obtained for use in equation (5).

It should be noted that the restoring moment of the torsion spring k affects the period of the oscillation (equation (6)) but does not affect the damping (equation (3)). It is possible, therefore, to adjust the period to any desired value without affecting the measurement of C_{n_r}.

APPARATUS AND PROCEDURE

The investigation was carried out in the NACA free-flight tunnel with the apparatus shown in figure 1. The upper portion of the strut to which the model is attached is mounted in ball bearings and is free to rotate within the fixed base. The model is therefore free to yaw but is restrained in roll and pitch. The movable portion of the strut is hinged to permit adjustments in the angle of attack of the model being tested.

A torsion spring connecting the fixed and movable portions of the strut provides the additional restoring moment necessary for obtaining short-period yawing oscillations. It is important that the period of the oscillations be fairly short to insure a well-defined oscillation envelope and therefore to permit an accurate measurement of damping.

The airplane model used in the tests is shown in figures 1 and 2. The wing of the model had an aspect ratio of 6.7 and a taper ratio of 0.40, and was equipped with partial-span split flaps deflected 60°. Two vertical tails, shown in figure 2, were used on the model. The model was mounted on the strut with its center of gravity on the axis of rotation.

The rectangular wing used in the investigation had an aspect ratio of 6 and an NACA 103 airfoil section. This airfoil section was used because of its high maximum lift coefficient at the low Reynolds numbers of the free-flight-tunnel tests. For some of the tests the wing was fitted with a split flap 20 percent of the wing chord and 60 percent of the wing span.

The airplane model was tested at dynamic pressures of 1.9 and 4.1 pounds per square foot. No appreciable change in C_{n_r} was noted with variation in dynamic pressure. The rectangular wing was tested only at a dynamic pressure of 1.9 pounds per square foot because of excessive vibration of the wing at higher values of dynamic pressure.

The testing procedure consisted simply in yawing the model approximately 10°, releasing it, and recording the resulting oscillations with a motion-picture camera mounted on top of the tunnel.

The friction of the system was determined from tests at zero airspeed with the models replaced by flat lead weights on long rods. These weights were adjusted to simulate the yawing moments of inertia of the models and were alined with the plane of rotation to give negligible air damping. In tests of the airplane model at zero airspeed with vertical tail removed, essentially the same damping was obtained as in the friction tests. It appeared, therefore, that a tail-off run at zero airspeed could be satisfactorily used to replace the special friction tests with lead weights.

The peaks of the oscillations recorded by the camera were read from the film record and plotted against time. The natural logarithms of the faired peaks were then plotted against time and the slopes of the resulting straight lines were graphical representations of the logarithmic decrements a and a_f. The numerical values for a and a_f were determined from the slopes by equation (2) and these values were substituted in equation (5) to obtain C_{n_r}.

Lift and drag coefficients and yawing-moment coefficients due to sideslip were determined by tests on the six-component balance in the tunnel for use in correlating the measured values of C_{n_r} with the theoretical derivatives.

THEORETICAL DAMPING DERIVATIVES

The value of C_{n_r} for a complete airplane may be assumed to be made up of directly additive contributions of the vertical tail, wing, and fuselage, if interference effects are neglected; that is,

$$C_{n_r} = \Delta C_{n_r(tail)} + \Delta C_{n_r(wing)} + \Delta C_{n_r(fuselage)}$$

It can be shown that the contribution of the vertical tail is

$$\Delta C_{n_r(tail)} = -2\frac{l}{b}\Delta C_{n_\beta(tail)} \qquad (8)$$

For a wing without flaps

$$\Delta C_{n_r(wing)} = K_0 C_{D_0} + K_1 C_L{}^2$$

Simple integration for K_0 yields

$$K_0 = -0.33\left(\frac{1 + 3\lambda}{2 + 2\lambda}\right) \qquad (9)$$

Values for K_1 are given in figure 13 of reference 2, which may be represented by the equation

$$K_1 = -0.031\left(1 - \frac{A - 6}{13} - \frac{1 - \lambda}{2.5}\right) \qquad (10)$$

The value -0.031 is for a rectangular wing of aspect ratio 6.0. Glauert, in reference 1, gives a value of −0.024 for this condition.

For a wing with partial-span flaps extended, the profile-drag term $K_0C_{D_0}$ becomes

$$K_0C_{D_0} = K_0C_{D_{0_w}} + K_f\Delta C_{D_{0_f}} \tag{11}$$

where

$$K_f = -0.33\left(\frac{b_f}{b}\right)^3 \frac{4 - 3\frac{b_f}{b}(1 - \lambda)}{2 + 2\lambda} \tag{12}$$

and the induced-drag term $K_1C_L^2$ takes the form

$$K_1C_L^2 = K_1C_{L_w}^2 + K_2\Delta C_{L_f}C_{L_w} + K_3\Delta C_{L_f}^2 \tag{13}$$

Values for K_1 and K_3 are given in figures 12 and 13 of reference 2, but the value for K_2 is not given in this reference and is apparently not available from other sources. Inasmuch as $\Delta C_{n_{r(wing)}}$ for the flaps-extended condition cannot be computed without the value of K_2, it appears desirable to prepare additional charts for this factor.

Calculations of reference 5 indicate that the theoretical value of $\Delta C_{n_{r(fuselage)}}$ is zero for fuselages that are ellipsoidal in shape.

RESULTS AND DISCUSSION

Contribution of Vertical Tail to C_{n_r}

Values of C_{n_r} for the complete model with partial-span flaps extended are given in figure 3 as a function of vertical-tail size. Values of $\Delta C_{n_{r(tail)}}$ are obtained directly from the data in figure 3 by subtracting the value of C_{n_r} with tail off from the values of C_{n_r} with tail on. The line drawn on the figure was computed from equation (8) and was based on the measured values of $\Delta C_{n_{\beta(tail)}}$ given in the table on figure 2. The agreement between the test points and this computed line is an indication that the contribution of the vertical tail to C_{n_r} for a midwing airplane can be computed with reasonable accuracy from the theoretical relation given in equation (8). For high- or low-wing airplanes a correction factor might be necessary for this relation because the sidewash at the tail, which varies with wing position, causes different changes in $\Delta C_{n_{\beta(tail)}}$ and $\Delta C_{n_{r(tail)}}$.

Contribution of Fuselage to C_{n_r}

The data of figure 4 show the variation of C_{n_r} with lift coefficient for the fuselage-wing combination and for the rectangular wing with partial-span flaps extended. The value of C_{n_r} for the fuselage and wing varied from -0.014 to -0.028 over the lift range covered in the airplane model tests.

A comparison of the C_{n_r} values for the fuselage-wing combination with the values for the rectangular wing with flaps extended (fig. 4) indicates that the fuselage had a negligible effect on C_{n_r}. Although it appears from a direct comparison of the data that the fuselage slightly reduced C_{n_r}, this apparent reduction was probably caused by the difference in plan form and by the greater profile drag of the rectangular wing. Other recent tests, the results of which are unpublished, have indicated values of $\Delta C_{n_{r(fuselage)}}$ ranging from -0.003 to -0.006. It appears, therefore, that the fuselage contribution to C_{n_r} is normally small enough to be neglected.

Contribution of Wing to C_{n_r}

Variation of $\Delta C_{n_{r(wing)}}$ with lift coefficient.- The data of figure 5 show that $\Delta C_{n_{r(wing)}}$ for the wing with flaps retracted varied as the square of the lift coefficient, as predicted by theory, but that the value of K_1 was smaller than the value predicted by either reference 1 or reference 2. The experimentally determined value of K_1 for the rectangular wing was -0.020; whereas reference 1 predicted a value of -0.024 and reference 2, a value of

-0.031. It appears that the value of -0.031 given by reference 2 and used in equation (10) is too large and should be replaced by -0.020.

The variation of C_{n_r} with lift coefficient for the wing with partial-span flaps extended (fig. 4) differed from the variation with flaps retracted in that the minimum value of C_{n_r} was obtained at a small positive lift coefficient rather than at zero lift. This result, which is also indicated by equation (13), is due to the fact that at zero lift the center flapped section is developing positive lift, the tip section is developing negative lift, and both are contributing to C_{n_r}. Inasmuch as no calculated value for the constant K_3 was available, no correlation of the theoretical and experimental variation of $\Delta C_{n_r(wing)}$ with lift coefficient could be made for the flaps-extended condition.

<u>Variation of $\Delta C_{n_r(wing)}$ with profile drag.</u>- The value of C_{n_r} for the wing with flaps retracted at zero lift was about -0.007, as shown in figure 5. The profile-drag coefficient C_{D_o} for the wing, as measured on the balance in the tunnel for the same condition, was 0.024. From these two values, K_o is found to be $\frac{-0.007}{0.024}$ or -0.29. Equation (9) yields 0.33 as the theoretical value for K_o for a rectangular wing. It appears that the calculated and the experimentally determined values of K_o are in fairly good agreement.

With the partial-span flaps deflected on the rectangular wing, the value of C_{n_r} due to profile drag can be obtained from the value of C_{n_r} at the lift coefficient given by the flap. For the wing tested, the flap gave an increment of lift coefficient of 0.60. From figure 4 at a lift coefficient of 0.60 the value of C_{n_r} was -0.017. Combining equations (11) and (13) and eliminating terms containing C_{L_w}, because $C_{L_w} = 0$ at $C_L = 0.60$, gives

$$C_{n_r} = K_o C_{D_{o_w}} + K_f \Delta C_{D_{o_f}} + K_3 \Delta C_{L_f}{}^2$$

The value $K_0 C_{D_{0_w}}$ was -0.007 from the wing-alone test; $\Delta C_{D_{0_f}}$ was 0.080 from force tests; K_f was -0.072 from equation (12); and K_3 was -0.0092 from reference 2. Then, for a value of ΔC_{L_f} of 0.60,

$$C_{n_r} = -0.007 + (-0.072\times0.080) + \left[-0.0092\times(0.60)^2\right]$$

$$= -0.007 -0.006 -0.003$$

$$= -0.016$$

This result is in good agreement with the measured value of -0.017. The magnitude of all of these factors is small, however, compared with the contribution of the tail surface.

Determination of C_{n_r} for Complete Model

The following empirical formula, which was derived from test results, should give a fair approximation of the value of C_{n_r} for a conventional midwing airplane:

$$C_{n_r} = -\left(2\frac{l}{b}\,\Delta C_{n_{\beta(\text{tail})}}\right) - \left[0.33\left(\frac{1+3\lambda}{2+2\lambda}\right)C_{D_0} + 0.020\left(1 - \frac{A-6}{13} - \frac{1-\lambda}{2.5}\right)C_L^2\right]$$

CONCLUDING REMARKS

The free-oscillation method of determining damping in yaw is considered very satisfactory in that it provided reasonably accurate results quickly and easily. The following conclusions were drawn from the results of the free-oscillation tests of the midwing airplane model and the rectangular wing model:

1. The experimental values for the yawing moment due to yawing contributed by the vertical tail were in good agreement with the calculated values.

2. The values of the yawing moment due to yawing contributed by the wing varied as the square of the lift coefficient but were lower than those predicted by theory.

3. The value of the yawing moment due to yawing contributed by the profile drag of the wing was approximately the same as the theoretical value.

4. The contribution of the fuselage to the yawing moment due to yawing was negligible compared with the value for the complete model.

5. The test results indicated that a fair approximation of the value of the yawing moment due to yawing for a conventional midwing airplane could be obtained from an empirical formula.

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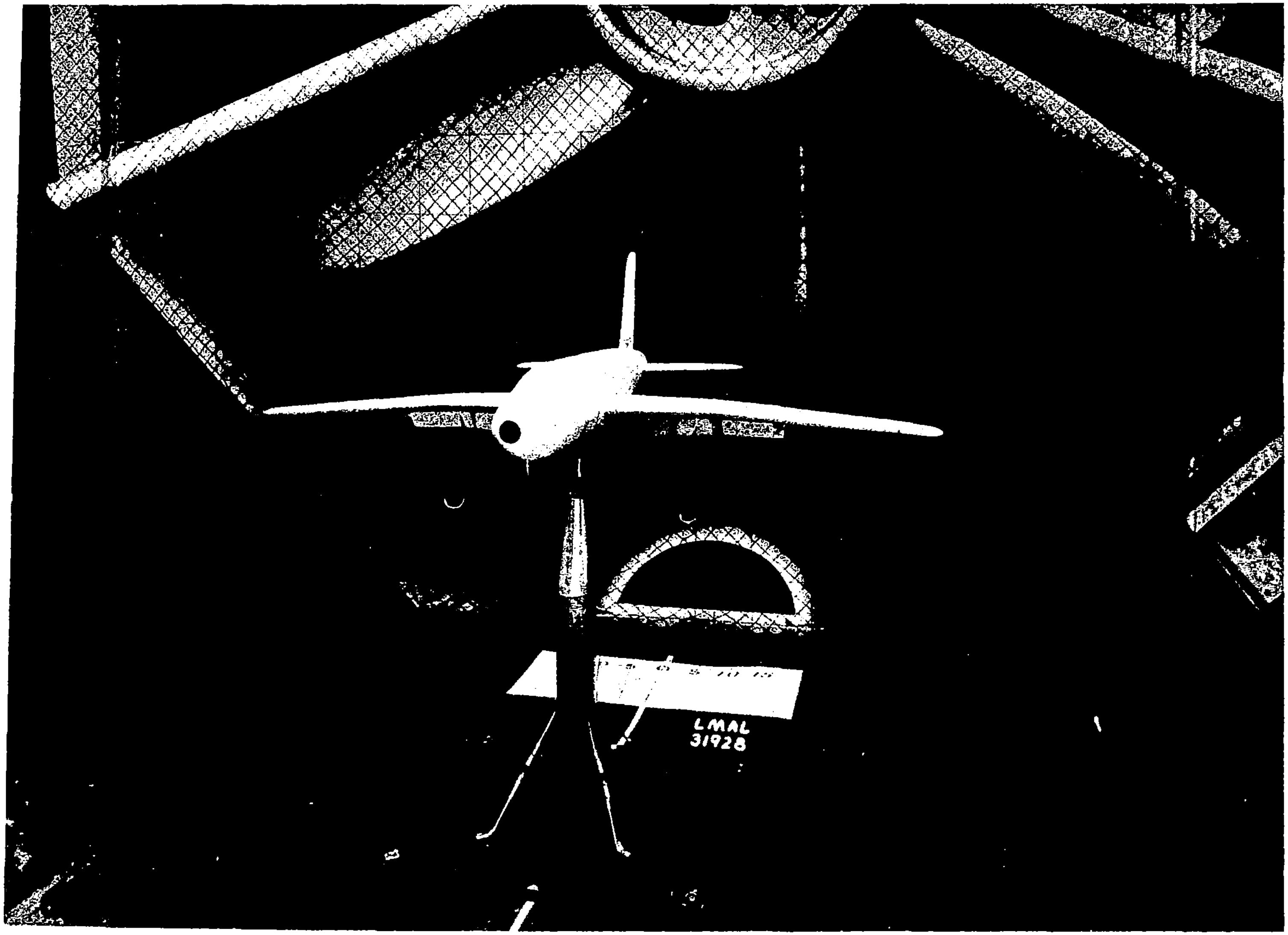

Figure 1.- Midwing airplane model mounted on yaw strut for damping tests in the NACA free-flight tunnel.

Tail	A	Area (percent S)	ΔC_{n_β}	l/b
1	1.9	5.2	0.07	0.35
2	1.9	10.5	.14	.35

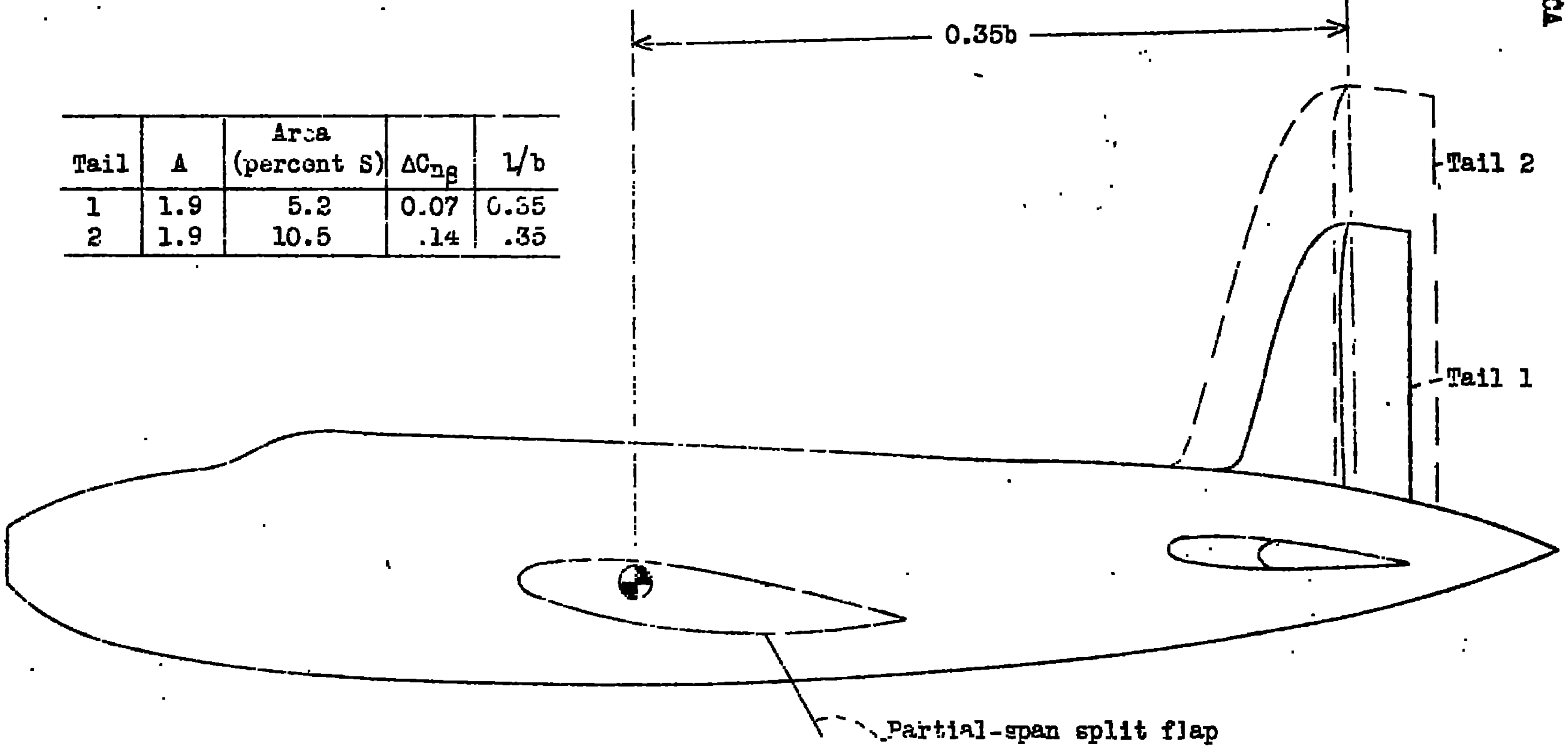

Figure 2.- Side elevation of model used in damping tests in NACA free-flight tunnel.

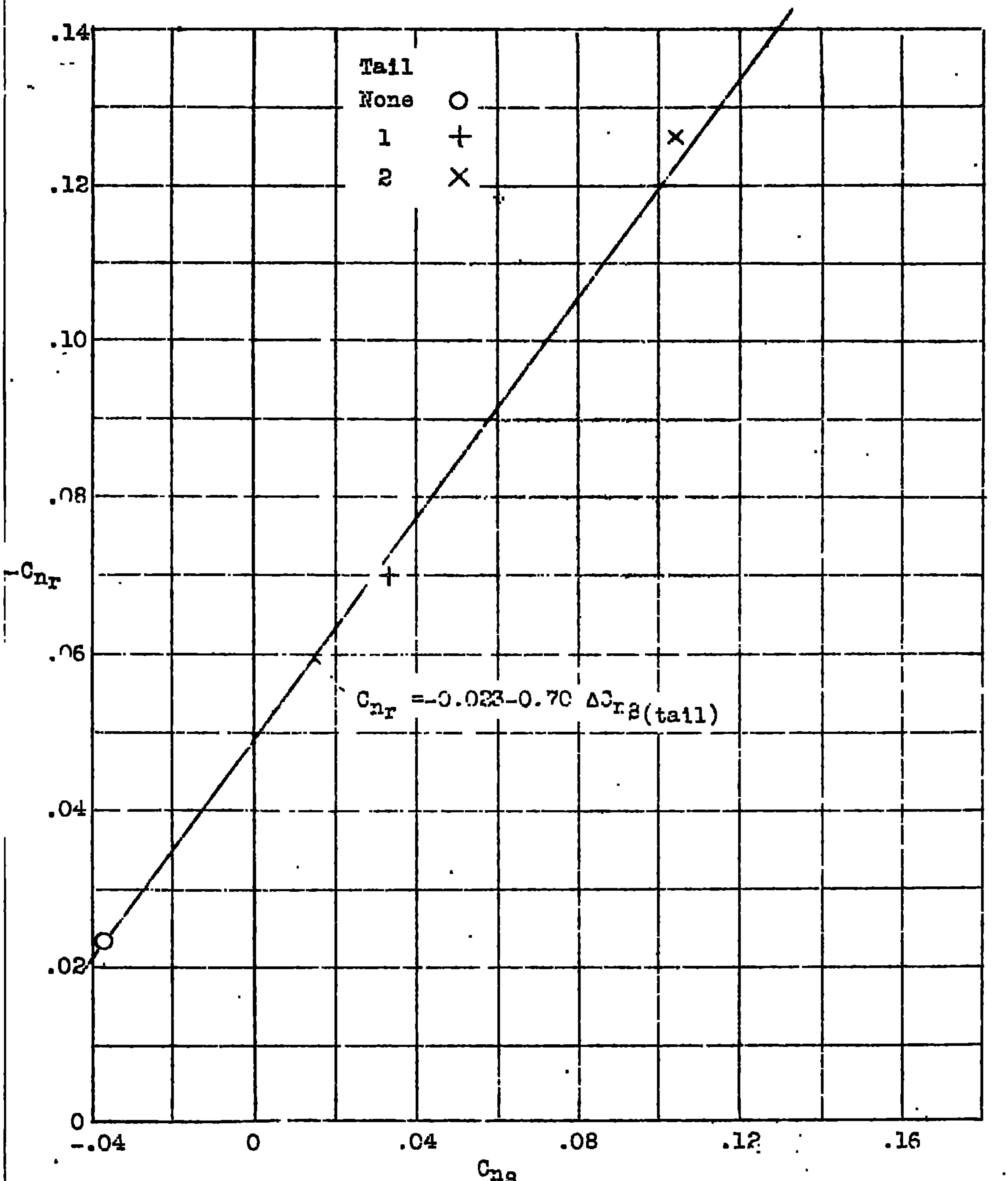

Figure 3.- Variation of damping in yaw with vertical-tail effectiveness. Midwing airplane model; flaps extended; C_L = 1.0.

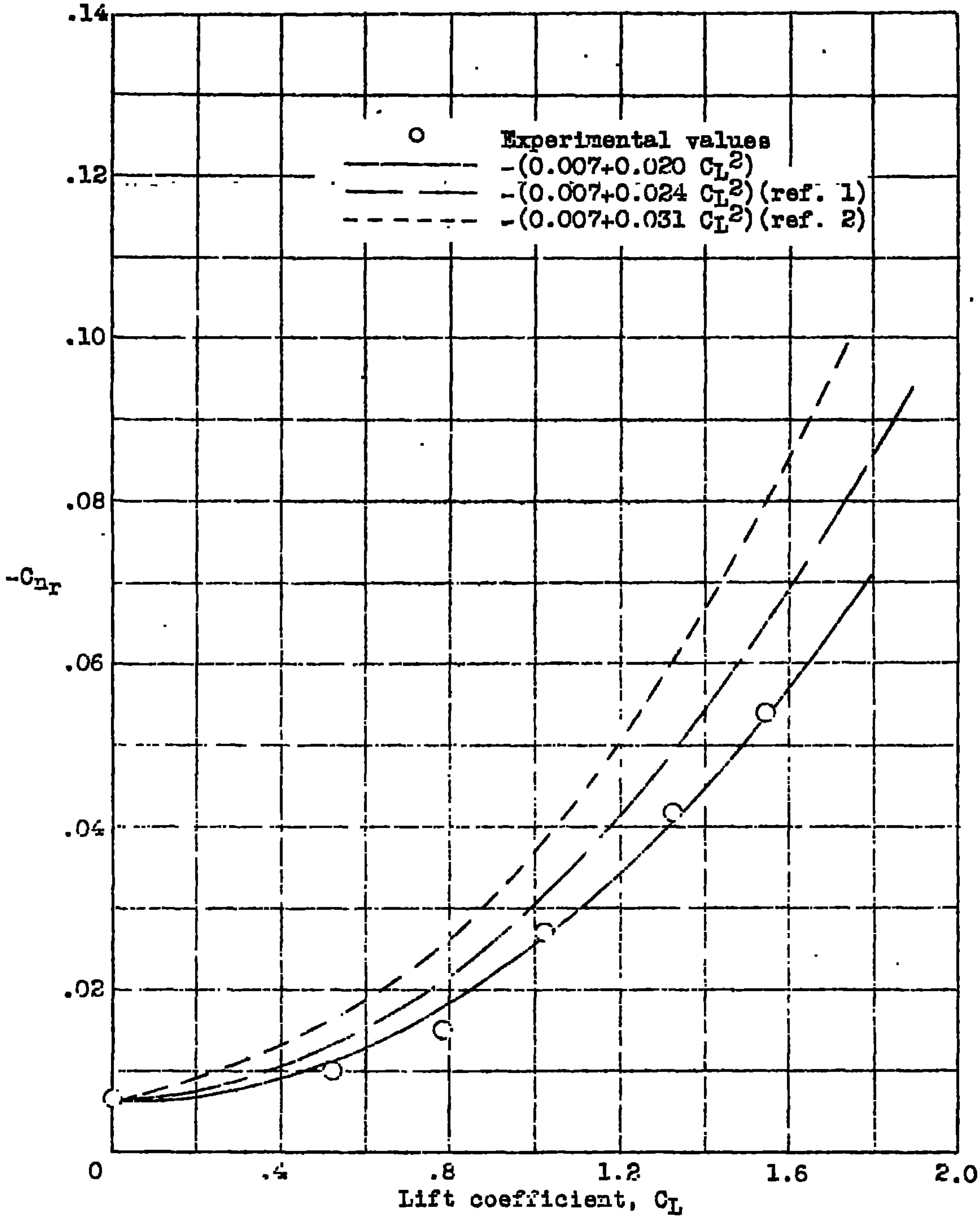

Figure 5.- Variation of damping in yaw with lift coefficient. Rectangular wing; flaps retracted.